RISK AND UNCERTAINTY: FROM ATLANTIC TO PACIFIC AS WELL AS FROM TECHNOLOGY TO FINANCE

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Keywords: Risk and Uncertainty, Commensurability of Risk and Return, Risk Management, Risk Value of Money, Quantitative Measurement and Economical Evaluation of Risk, Financial Management, Investment Appraisal, Investment Management.

1. **Introduction**. Permanent changes are the most widespread in the state of nature in our time. These changes stipulate permanent non-coincidences between the expected and being happened, which challenge decision making under risk and uncertainty.

Non-deterministic nature of the future in the main, as well as processes of unification and globalization, provocate the rapid changes, which occur in all spheres of humanities' life as well as in all geographical latitudes, longitudes and altitudes. These would be one of the typical features at the beginning of the 21st millenium. Today's society ought to prove to be prepared for the acceptation of these changes. However, risk and uncertainty, which accompany the changes, appears to be the main obstacle for adequate reaction to these changes.

Much of today's scientific research results, as well as historical date analyses are formulated in a deterministic (under certainty) manner. Firstly, it is the true of researches and investigations in the areas of economics, demography, technology, finance, etc. At the same time it is essential for decision making to have probabilistically structured information in order to choose the most suitable relation between risk and return of activity according to ones own utility.

This paper deals with the problems of searching for the possibilities of commensurability risk and return, which is vitally important in investment management theory and practice. The objectives of the paper are intended to be achieved through the following reasoning:

- Classical approach to risk and return commensuration for an investment is based on defining equivalency relationship between the scope and the variability of return of the investment. The classical approach is needed for statistical data representing the distribution of returns and information about impact of separate risk factors on the total risk. Perfect realization of this intention is impossible without constructing and the employment of causal models for risk and return commensurability (*CMRRC*). Some ideas of working out such models are proposed in this paper.

- Non-adequacy of everyday practice in financial, investment etc. decision making and in corresponding conceptual and quantitative modeling with the probabilistic nature of future events is evident even in methodical and scientific literature and contemporary software. Some practical steps for taking away these non-adequacies are proposed in the paper.
- Traditional investment appraisal methods are based on discounted outlays and inflows on the investment. By its nature discount rate should be treated as a stochastic parameter, i. e. defined by its probability distribution. This presumption ought to be used for quantitative assessment of discount rate risk impact on investment appraisal probability distribution. The concept of risk money value should be used.
- Territorial mappings of regions and countries according to risk levels usually are prepared without consideration of money risk value. Some conceptual problems and academic examples are discussed in this paper.

2. Management under the risk or risk management?

2.1. Understanding of management and investment management. If *management* is understood as an activity for the effective use and co-ordination of resources such as intelligence, capital, plant, materials and labour to achieve defined objectives with maximum efficiency [3], then *financial management* must be treated as an activity for effective use and co-ordination of financial resources. Since investment as a branch of finance could in a general sense be treated as any employment of capital in expectation of gain, whether in a business, farm, urban real estate, bonds, stocks, merchandise, education etc. [2], then *investment management* is management of capital employment for future gains.

In reality, for complex spheres of business activity or analogic environments of these activities such as finance, investment, risk and others problem of sophisticate of conception of management reveals itself. It can be seen that because at this sophisticate the reference books, encyclopaedias and even text books do not give an interpretation of such compound categories like “finance management”, “investment management”, “risk management” and so on, even if the composition of such categories are included in these titles. It is obvious that it is unclear how to use together these complex categories such as finance and management or investment and management or risk and management or more formally speaking, how to define intersection of these categories.

2.2. Changes, risk and uncertainty in investment management. The phenomenon of risk or uncertainty or risk or incomplete information plays a pervasive role in economic life. The most fundamental distinction in this branch of economic theory and the origin of discussion if ‘risk versus uncertainty’ or ‘risk or uncertainty’ is due to Knight [9]. A situation is said to involve *risk* if the randomness facing an economic agent can be expressed in terms of specific numerical probabilities. On the other hand, situation where the agent cannot (or does not) assign actual probabilities to the alternatively possible occurrences are said to involve *uncertainty* [1]. These two different but complementary conceptions of arrangement of information for decision making have amounted to complete fields of theory with active practical exploitation of this knowledge.

Webster’s dictionary defines risk as “the chance of injury, damage or loss”. However, risk should not be confused with chance (or probability) since it is an amalgam of both this probability and size of event. If the 1 in 10 chance of a loss is one of making a very big loss indeed, this can be a more risky event than one where the probable is exactly the same but the risk of loss, while still 1 in 10, is much smaller. This is one of many occasions for saying risk management, instead of saying management under the risk.

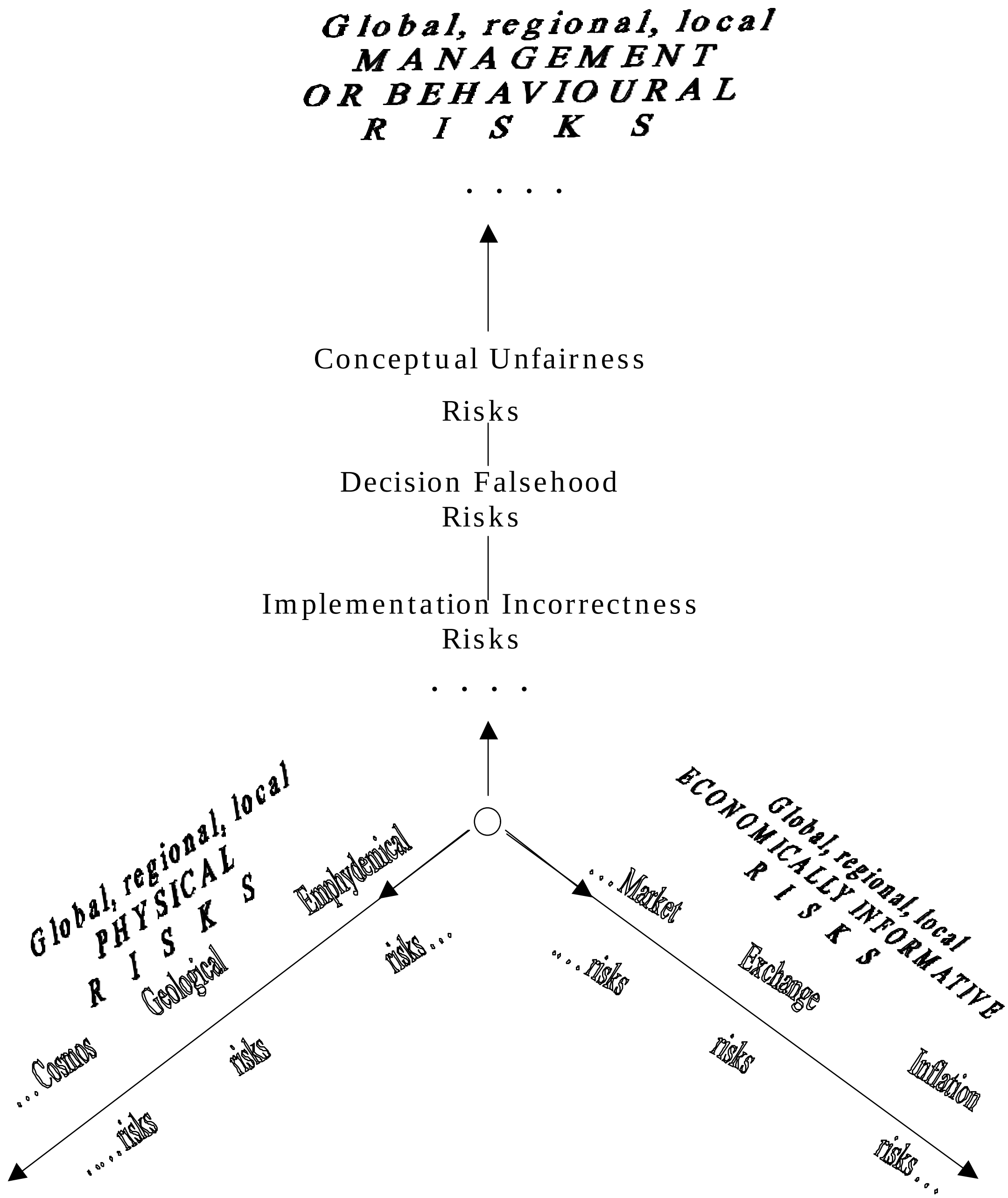
Such a manner of understanding risk requires a further glance into Webster’s dictionary where ‘loss’ is explained not only as a fact or act, or process of losing, but as a failure to keep, maintain or the use of opportunities. Of course, business losses usually are measured by the amount or degree of failure to use opportunities completely. So, the risk is the quantifiable likelihood of loss or less-than-expected returns.

Because non coincidence of the happened with the expected is a precondition of failure to use the opportunities i. e. if having happened is even better than expected, but the possibility of failure to use the opportunities is positive, then everyone should be convinced that variance or standard deviation is a more adequate or more exhaustive measurement of risk than semivariance.

Attitudes to risk vary, depending very much on the psychological make-up of the risk taker and the probable outcomes. Decision making when all the real possibilities of outcome are considered makes this process more interactive with inner potentiality of the risk taker. Because of presence of permanent possibility of high losses to the investor from one hand and possessing developed quantitative methods of management from the other hand insist on saying ‘risk management’ instead of saying ‘management under the risk’.

Practically, there are no investments without risk. Even the investments that appear to be completely risk free are exposed to some kind of risk. For example, Treasury securities are considered to be the safest investment. They are free of default risk but they are not free of interest rate risk. These risks can receive a huge variety of forms: from financial up to psychological and even more unexpected forms of risk. In the case of risk of investment management an extreme attention must be paid to quantitative measuring and economical assessment of these risks.

2.3. The schemes of risk structurisation. There are a great variety of risks as well as schemes of its structurisation. One of them, presented in scheme 1, is called forth by intention to remind about the existence of dual information – physical and economical or about the possibility to organise economical equivalency for the processes occurring in the physical world. Practically, decision making in many situations is based on economical equivalency rather than on original physical information.



Scheme 1. A. Glimpse at Risk Structures

Risk reception is very individual for different subjects, i. e. different business entities or different kinds of activities. Therefore, when the subjects differ from one another the pool of total risk factors or risk types vary from one list to another, as well the scheme of its structurisation also changes. For example, in the case of investment activity these factors investigated by many researches are: default risk, interest rate risk, market risk, management risk, purchasing power risk, marketability risk, political risk, collability risk, convertability risk.

3. Commensurability of Risk and Return.

3.1. The analytical models of risk and return interdependency. To understand the risk, the investor has to understand the different types of risk as well as to have a comprehension about quantitative measuring and economic assessment of the risk. The nature of dependency of return on different factors, that usually are effected by many circumstances and which are exposed to risk, needs to exploit the stochastic form of the relationship between return and risk. Consequently, return and risk ought to be treated like separate random variables and dependency risk of return on the scope and variability of factors such as rate of inflation, rate of exchange etc. must be studied in a visible (explicit or implicit) manner.

While definitive results of evaluation of the future investment (project) performance could be expressed in a variety of terms: in the years of payback, in the amount of dollars of net present value (*NPV)*, in percent of internal rate of return (*IRR)*, in decimals of total return etc. these indicators could never appear in the form of point estimation. They ought to appear in the form of distribution of random variables. Meanwhile, classical methods of risk return relationship analysis, i. e. the arbitrage pricing theory (*APT*), the capital market line (*CML*), capital asset pricing model (*CAPM*), and even the characteristic regression line (*CRL*) present dependency of the expected return on scope of risk (or many kinds of risks) in the deterministic manner.

Furthermore, these methods of risk-return relationship analysis are directed at commensuration of the expectation of rate of return of an investment with the riskiness of the return. Meanwhile, riskiness of return originates on the basis of non-stability of such financial parameters like interest rate, inflation rate, exchange rate etc. These circumstances compel risk – return dependency researchers to define causalities and select functional dependencies between probability distributions of return on an investment and variability of return causing risk factors. At that time, it must be stated that the main indicators of investment efficiency, like rate of return, payback and other, have explicitly or an implicitly definable functional dependency upon the above mentioned parameters.

In the next paragraph, the discussion will continue on the scantiness of classical risk-return analysis methods for investigation quantitative dependency between return on investment and volatility of return bearing factors variability altogether with presenting analytical models providing possibilities for quantitative commensuration of return on investment and variability of factors.

3.2. Classical approach to risk and return. In this paper the classical approach to risk-return relationship analysis will be understood as a certain ideology. Traditional methods of investment appraisal: i. e. Return on capital employed (*ROCE*), Payback method (*PB*), Net present value (*NPV*), Discounted cash flow (*DCF*) and Internal rate of return (*IRR*) as well as employment of Arbitrage pricing model (*APM*), Characteristic regression line (*CRL*) model, Capital market line (*CML*) model, Capital asset pricing model (*CAPM*) for commensuration of risk and return, are based on such ideology for risk and return relation analysis.

When summarizing 3.1 chapter and the great amount of literature on investment appraisal and commensurability of risk and return on investment that individually could be represented by [4] and [5], the following conclusions could be drawn:

1. Classical models for the defining relationship between return on investment and risk are based on non-stochastic ratio between rate of return and variability of this rate and hardly can adapt itself for direct commensuration return on investment with an amount and level of risk factors.
2. Causal models for risk and return commensuration ought to help to determine a more precise estimation of impact of every risk factor as well as make quantitative arrangement of these factors.

3.3. Causal models for risk and return commensuration (CMRRC) Quantitative assessment and economic appraisal of the risk factors are required for having quantitative relationships between return on investment and risk indicators. Some of the primary risk factors i.e. quantitative indicators of these factors e.g. interest rate, discount rate, rate of exchange solely or even jointly enter models for defining *NPV*, *DCF*, *IRR* and many indicators presenting yield or return of investments or financial contract prices.

$$S_T = \frac{(1+i)^T - 1}{i}$$

For example:

where S_T = accumulated sum of one dollar,

T = time period for accumulation,

i = interest rate;

$$NPV = \sum_{t=1}^{T} \frac{I_t}{\prod_{i=1}^{t} (1 + d_i)}$$

where NPV = net present value,

I_t = net income in year t,

d_t =discount rate in year t;

$$PACO = p\,(ER, SP, DR_fR, FR_fR, V, T)$$

where $PACO$ = Price of American Currency Option ($),

ER = exchange rate ($ per Foreign),

SP = strike price ($ per Foreign),

DR_fR = domestic risk-free rate (% per year),

FR_fR = foreign risk-free rate (% per year),

V = volatility of exchange (% per year),

T = time to maturity (years).

Since discount rate, interest rate, exchange rate as well as many other financial parameters have the probabilistic nature, their behavior in future could be defined only by means of the probabilistic distribution of their possibilities. As a consequence of this and some other circumstances accumulated sum – S_T, net present value – NPV and American Currency Option Price – $PACO$ ought to be understood as stochastic variables possessing its own probabilities distributions. These distributions as probabilistic distributions of resulting variables are determined by type and parameters of distributions of independent variables, whose role in this case perform interest rate, discount rate and exchange rate. In many such cases for defining these distributions, it has been met with new technical problems. These problems have arisen because of the difficulties in making an assessment of the type and parameters of probability distribution of resulting variables, in our case S_T, NPV and $PACO$, according to type and parameters of independent variables – interest rate, discount rate, exchange rate.

For effective solutions of such problems computerized imitative techniques could be used successfully. Effectiveness of computerized imitative technique could be decomposed on to two main results. Firstly, in many occasions assessment of type and estimation of probabilities distribution parameters of resulting variables according to in advance defined probabilities distributions of primary factors, became too expensive or even impossible. Secondly, if the above mentioned assessment is performed by means of computerized imitation, it can be used not only for professionals of such kinds of activity, but also for ordinary users like economists, analysts of finance, managers etc. Massive use of the computerized imitative technique could create a new era in employment more adequate and, of course, a more sophisticated technique for decision making.

3.4. Simulation approach. Modern computers have made it both feasible and relatively inexpensive to apply simulation techniques to financial decisions as well as to assist theoretical researcher [6]. In the first case simulation is a planning tool that models some processes.

For example, if it is assumed that interest rates in domestic and foreign countries are non-random and constant, and changes in the exchange rate follow a lognormal probability distribution. Given these assumptions, the value of a European foreign currency call option is [7]:

$$c\ (S, T; r, r^*, EX) = S\exp(-r^*T)N_1(d_1) - EX\exp(-rT)N_1(d_1 - \sigma\sqrt{T})$$

where c = the value of European foreign currency call option,

S = strike price,

EX = the exchange rate (domestic/unit of foreign currency),

r = foreign risk-free rate of interest per unit time,

r^* = domestic risk-free rate of interest per unit time,

T = maturity of the option,

d_1 = [ln(S/EX + (r-r* + σ^2/2)T] / $\sigma\sqrt{T}$,

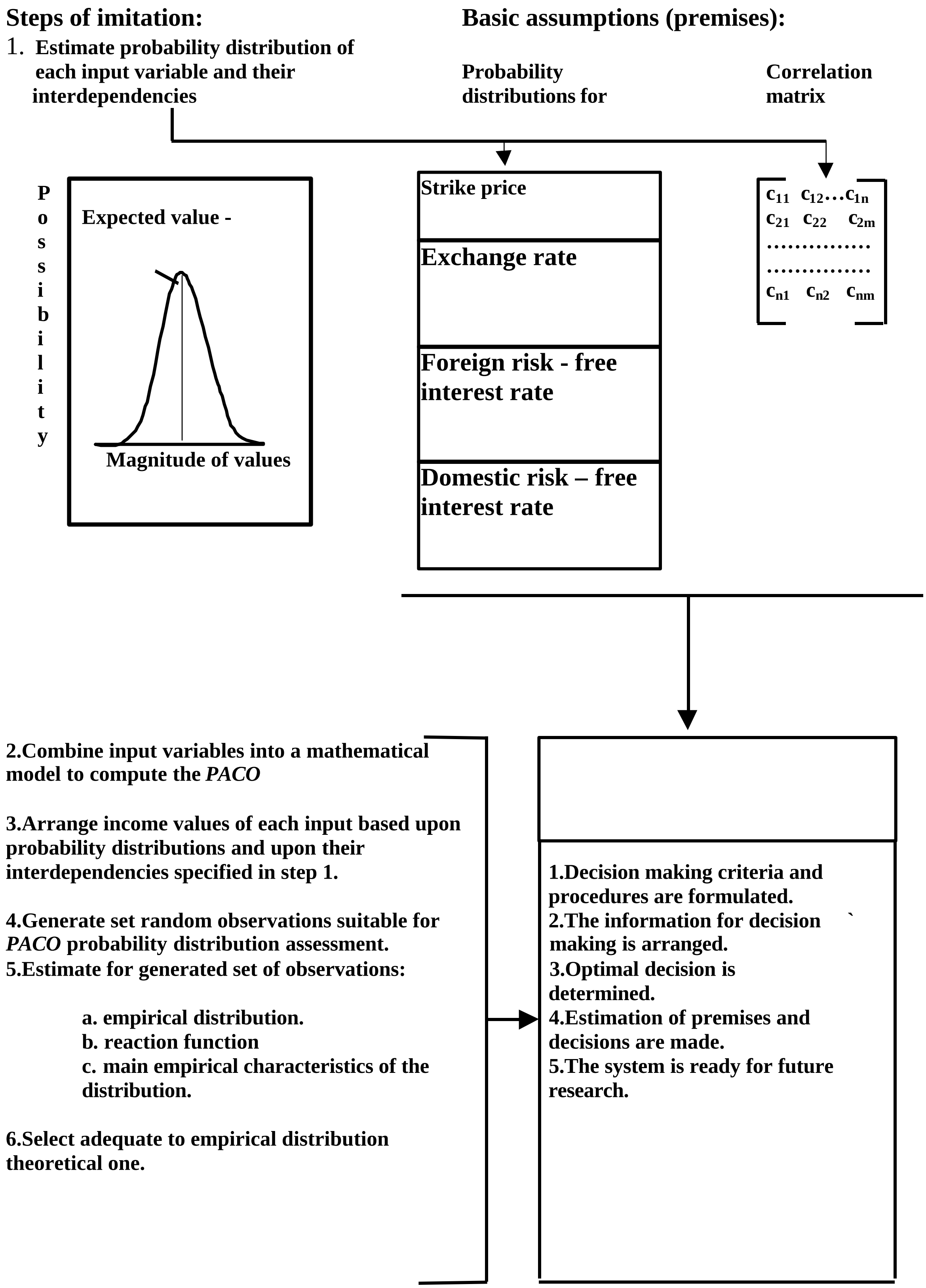
σ^2 = variance of the proportional change or return per unit time of the exchange rate,

N_1 = the cumulative normal distribution.

Of course, if this evaluation is valid for each separate quantity of interest rate, then it is valid for all the spectrum of possible quantities of interest rate as well as for all the spectrum of possible quantities of other variables under the presumption that each combination of quantities of different variables is rightful to be in the common equation. So, if we know the probability distribution of each variable as well as all interactions between individual variables and all kinds of multi-correlation, then the computerized imitative system can help planners or managers to define the probabilities distribution for foreign currency call option value.

In the second case when, for example, the probabilities distribution of option price is defined (observed or projected), and the validity of the model is assumed, it is possible “to calculate” i.e. to make approach probabilities distribution of the *implicit variance*. Of course, in many similar situations, practically, there are not any possibilities to determine probability distribution for implicit variables being included in sophisticated models. An illustration of the simulation approach used for practical calculations and estimations is given below (in *Figure 2*).

Figure 2. The illustrative scheme for imitative approach



model for implicitly defined variables. The approach gives a possibility to generate such an amount of observations, which satisfy the model restrictions when other variables involved vary throughout the entire set of its possible meanings. After the generation of the sufficient amount of "random" observations, theoretical distribution functions giving quite exact approximation on the "data" are developed [10].

4. The practical view of the probabilistic nature of the future possibilities

4.1. Risk value of money. When one is entering a treasure of knowledge from various sources of this knowledge on finance, real estate, insurance etc. or is trying to make this knowledge more complete and adequate to practice, usually, the same stands to meet him/her at the entrance. These and those stands are: accumulated sum of 1$ when interest rate i equals - 1%, 2%, …n% and time t changes – 1 year, 2 year, … T year; discounted quantity of 1$ when interest rate equals - 1%, 2%, …n% and T year; others tables about time and interest rate impact on different processes under consideration. Those and many similar tables accompany the reader through the entire text and certainly are placed at the last pages of manuals, handbooks, monographs and others issues, in use today.

Of course, if to be more precise in today's issues, the reader is being sent more and more often to contemporary calculators, where presentation of time impact on processes under consideration is made in a wider and deeper manner. But the ideology remains the same: presentation is being made on a great variety of possible outcomes, but these aren't perspectives of iteration between time and permanent changes, i.e. between time and non-deterministic nature of future, i. e. between changes in value and risk impact.

It is very important to understand, for example, in which manner to change the accumulated sum or discounted quantity of $1 in reality when intensity of risk changes. Everyone can confirm that the considered and mentioned tables do not answer to the question about character of iteration between time and the probabilistic nature of the future adequately.

If one could just recollect his or her imagination of the future and remind himself or herself about the impossibility to present the future as one point, i.e. as one variant or even as a spectrum of inter-exclusive variants, then the conscientious conclusion about behavior of interest rate in the future would be that in only a very successful case, the future of interest rate could be presented as the its probabilities distribution while in other cases it would be a situation under uncertainty. So, if the fact just stated is a reflection of reality, then the possibilities of accumulation similarly to the deterministic case - figure 3 could be presented as the distribution of possibility to accumulate, when time changes from (as in *Figure 4*) 1 to 30 years and rate of interest is presented as a set of normal distributed random variables with expected value of 10%, enlarging variation and decreasing correlation between annual interest rates.

If to compare the data situated in *Figure 3* and *Figure 4* then it could be argued undoubtedly that from the information situated in *Figure 3* there could not be a vision pictured as presented in *Figure 4.* When we need to answer the question, if the probabilistic presentation of future is more informative than the deterministic one (like in *Figure 3*), such items must enter into the answer:

- Contemporary decision making algorithms are oriented on probabilistically arranged information,
- Forecasting schemes and methods are not adapted to give point searching estimations of the future,
- Only after understanding what depiction of the future is given by data presented in *Figure 4*, some clarity of sense presented in *Figure 3* could be understood,
- The 2-nd column in *Table 1* presents by itself a new type of “upholsters”, i. e. the tables mentioned at the beginning of the chapter, i. e. possible outcomes of the accumulated sum in future years, which is presented by their probability distributions. In the mentioned column, abbreviations *N* and *LN* are used to note that normal or log-normal probability distribution are better suited in the case for a description of the corresponding data than other types of probability distributions. Symbols m_i and s_i^2 signify the expected value and variation of corresponding empirical distributions.

Figure 3

Relationship between accumulated value S of 1$ and period of accumulation when interest rate is 10%

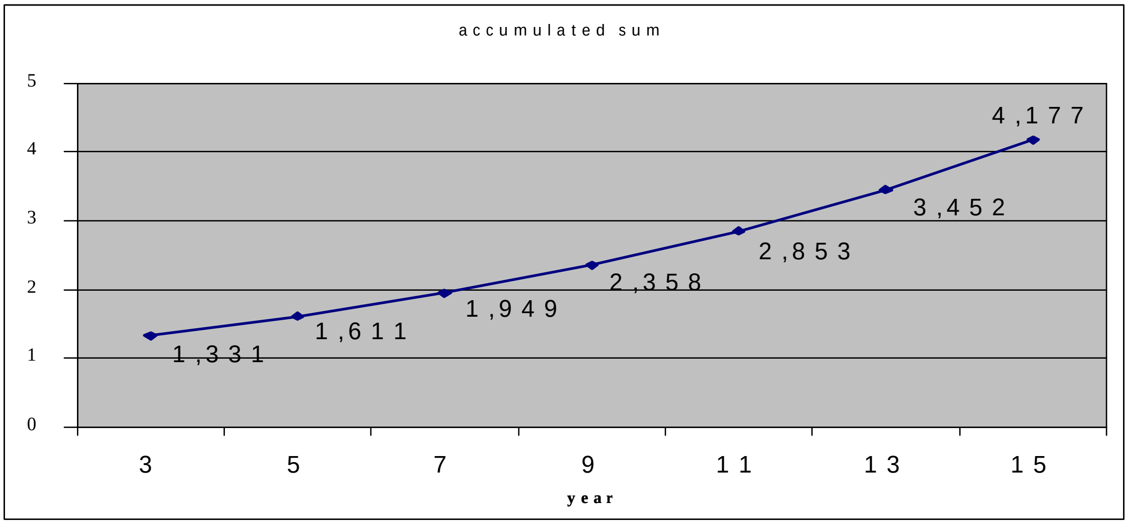
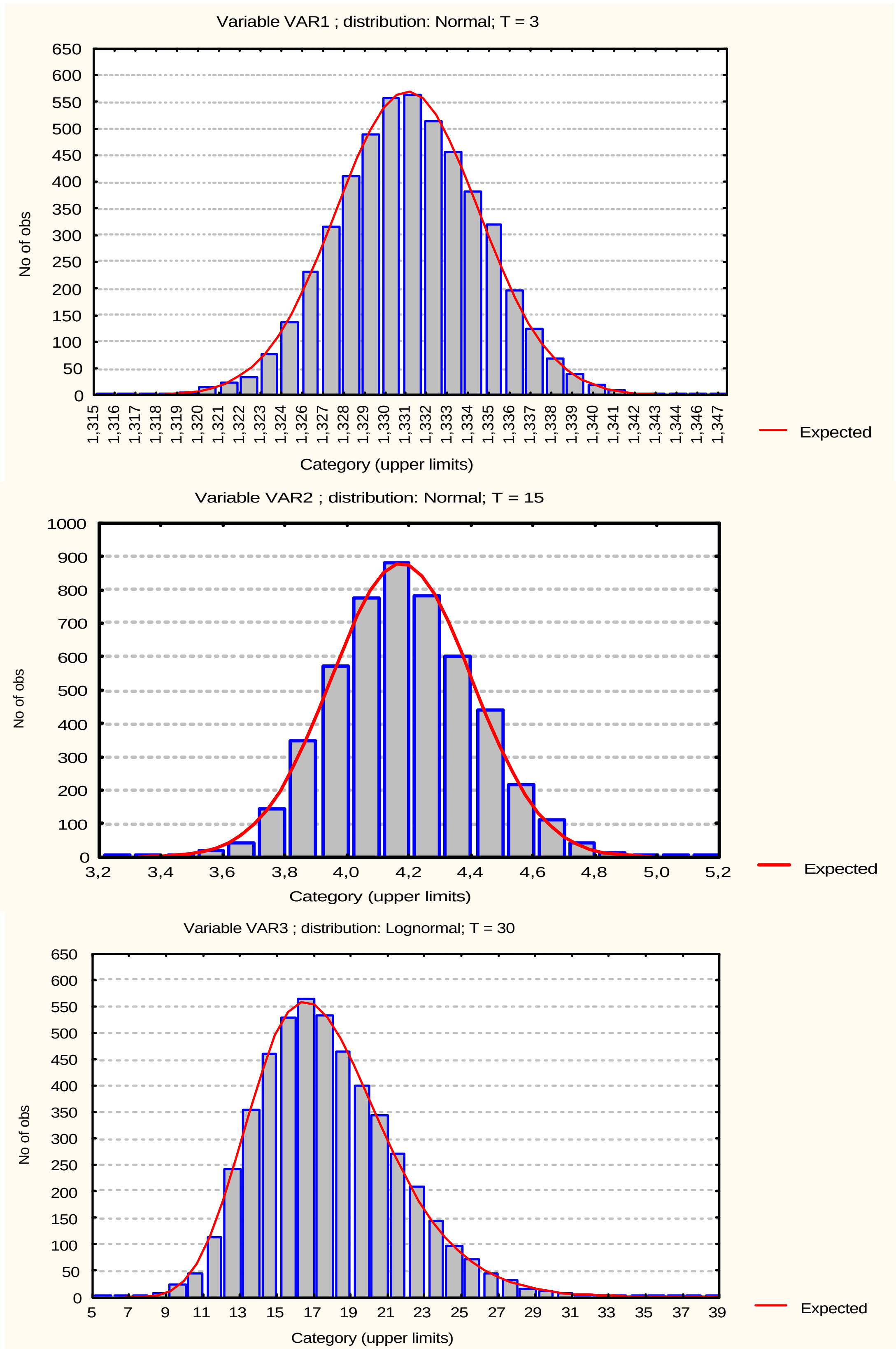


Figure 4

Probability distribution of accumulated value of 1$ in presence of different accumulation periods T_i and behavior of interest rate as it is described on page 15

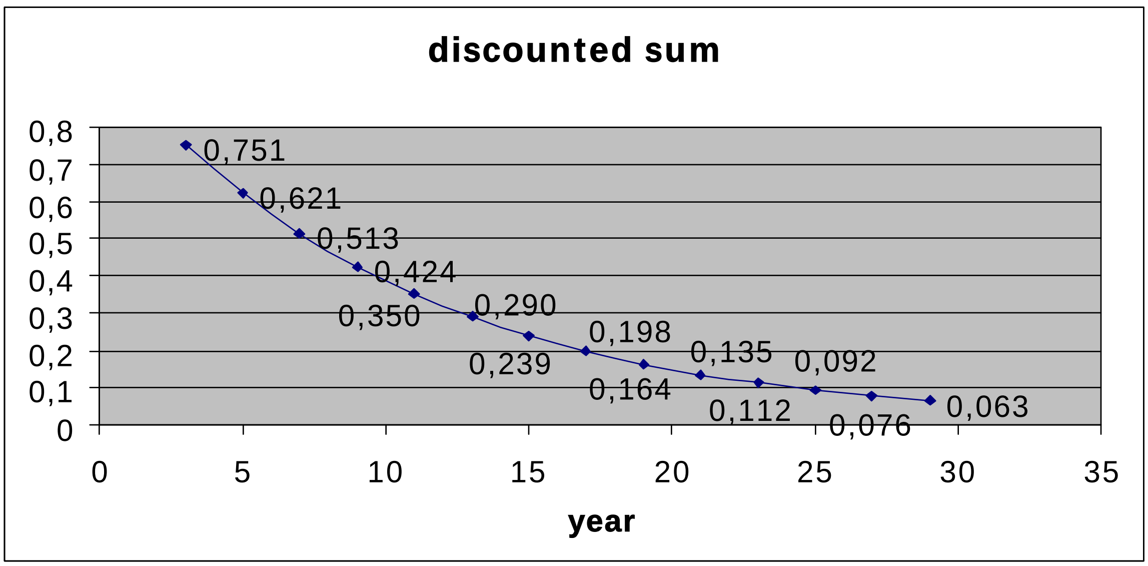


4.2. Some exploratory results of imitative approach to assessment of discount rate risk on investment appraisal

4.2.1. The simple case. Time impacts on future money value usually are being estimated by means of discounting future cash flows. The most simple and very often used tables for exposing time impact on money value are discounted 1$ value tables which are situated in every manual handbook and monograph on finance, real estate and etc. Presented values, where the discounted quantities of 1$, reflects the time impact, when time distance from present moment is growing and this is repeated for different discount rate quantities. Everyone knows these tables and should agree that neither these tables nor any dependency derived from this table (see for example figure 5) could depict impact of contingency on time value of money (see figure 6).

Figure 5

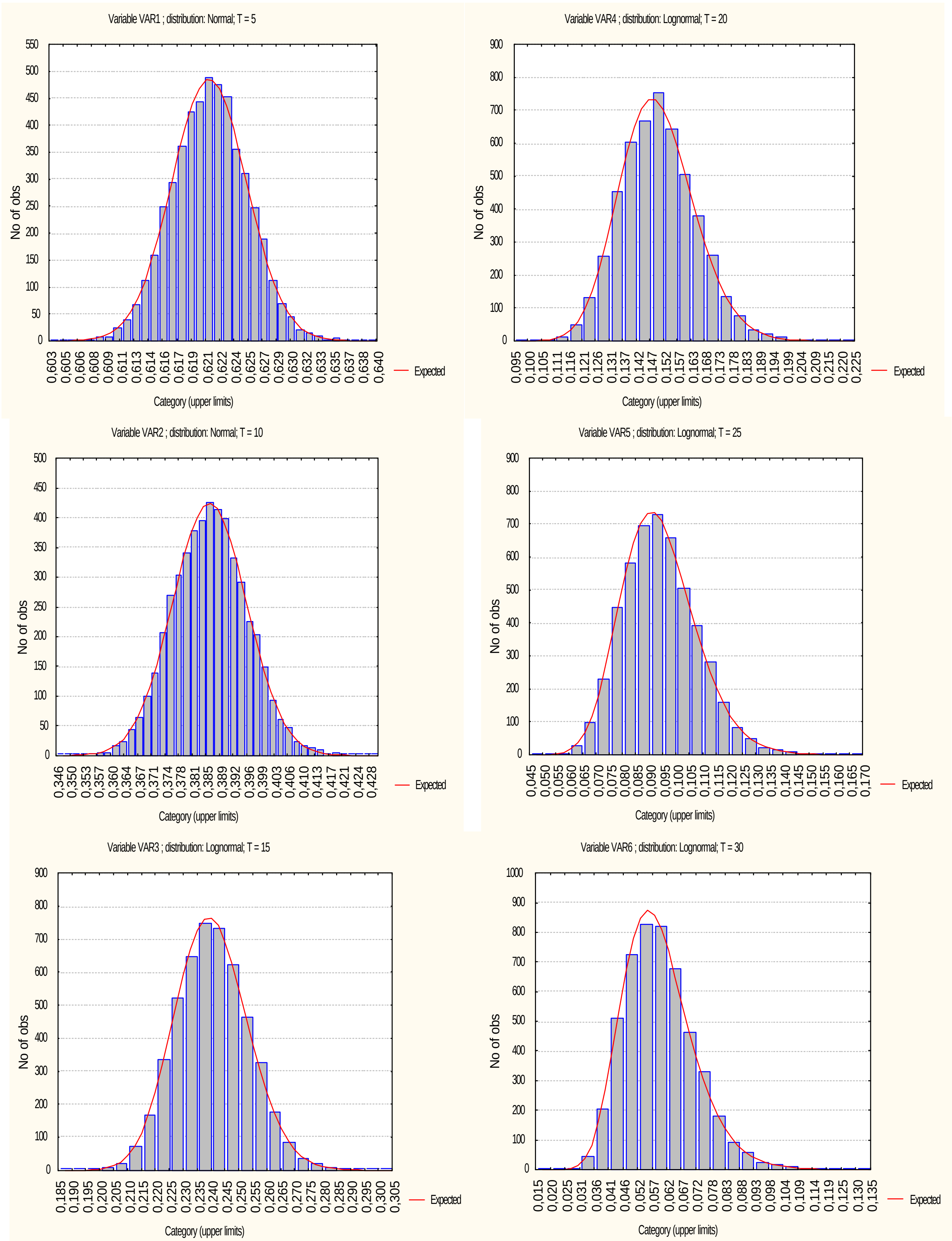
Relationship between discounted value S of 1$ and period of discounting when interest rate is 10%



Though, if we have supposed presumptions about the probabilistic nature of the future the above mentioned discounting tables fragment given in figure 5 cannot help us understand the way in which contingencies are performed in regularities. If, for example, it is presupposed that discount rate is defined by the normal distribution of probability, then joint impact of time and contingency on discounting could be depicted by the probability distributions of discounted value of 1$ when T = 5, 10, 15, 20, 25, 30 (see *Figure 6*).

Figure 6

Probability distributions of discounted value of 1$ in presence of different discounting periods T_i



Each of the cases presented in *Figure 6* urge for an explanation of the separate sense of each or joint effects. Variants 1 and 2 represent probabilities distributions of discounted quantities of 1$ value throughout 5 and 10 year. Both of these variants “obey” the normal type of distribution. But at the time interval, longer than 10 years, every case more and more explicitly, coincides with the log-normal type of the distribution. If it is presupposed that each cash flow and discount rate alters with another than the normal type of distribution, then the type of distribution changes more rapidly and unexpectedly.

4.2.2. Sophisticated cases. Of course, simpler or more sophisticated terms are used in the technical sense. More sophisticated means that for practical solution of models in which depending (resulting) variables can not obtain explicit expression more sophisticated technique have to be used.

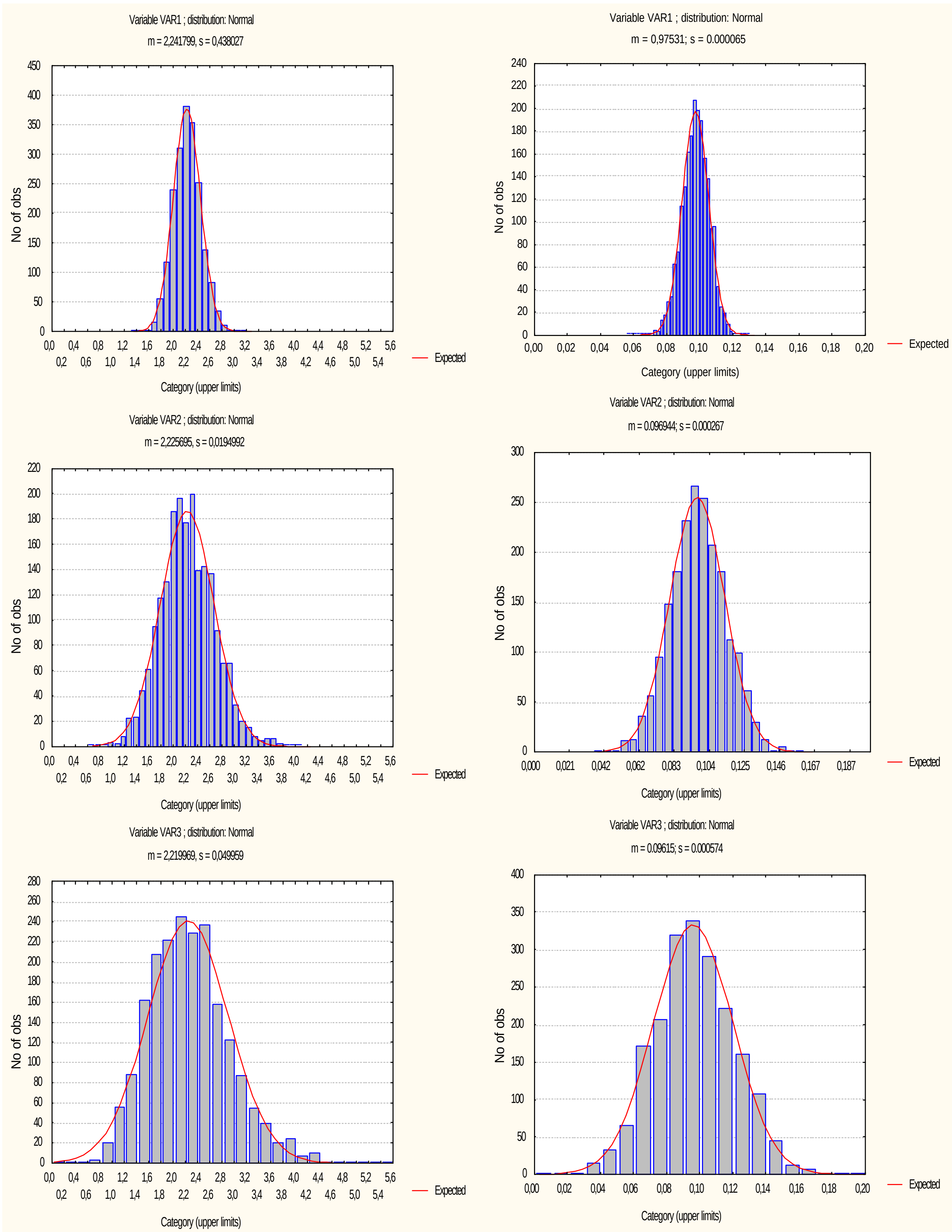
As it has been seen from the former chapter, more adequate presumptions about the nature of behavior of discount rates in the future allows one to obtain more adequate information about alteration of discounted quantities of 1$. Probabilistically arranged information more adequately reflects the possibilities of events under consideration and is more suitable for decision makers, i. e. more suitable to take into account his/her or its utility function.

However, we must remember that our primary objective was to look for possibilities to find dependency between definitive (dependant) results and independent factors. In our cases it should be *NPV* and *IRR* from one side and discount rate risk, i. e. variability of the interest rate measured by standard deviation, from the other side.

Let us consider *Figure 7*. In the *Figure 7* case *a.* there are distributions of *NPV* of discounted cash flow of - 10 mln. $ in the base year, and consequently 5 mln $, 5 mln $, and 5 mln. $ in the next three years when discount rate alters in accordance with the normal distributions: *N*(10%;1%) – in upper box, *N*(10%;2%) - in middle box, *N*(10%;3%) – in lower box. You can find that consequent probability distributions of *NPV* are: *N*(2,2418;0,0044), *N*(2,2257;0,0195), and *N*(2,2195;0,0499). So one can use defined distribution functions or even visually estimate probabilities of events in which you are interested. At the same way, you can estimate discount rate risk (variability) impact on probability distribution of *NPV*.

Figure 7

a. The probability distributions of *NPV* *b*. The probability distributions of *IRR*



In principal a technically more sophisticated case is situated in the *Figure 7* case *b*. As in the previous case, one can look into it by means of derived formulas or visually dependency of *IRR*, when factor of variety of the future income changes from *N*(1;0,1) in the first variant up to *N*(1;0,2) in the variant 2, and *N*(1;0,3) – in the variant 3. When estimating an impact of the risk (variability) of the income on probability distribution of *IRR* one can also observe that the type of the distribution has a tendency to change from the normal to the log-normal.

4.3. Psychological effects of probabilistic presentation of future

If to compare some calculations situated in *Figure 7* and *3*, for example defining accumulated sum of 1$ in 15 year period when interest rate equals 10% (as in *Figure 3*) and when behavior of interest rates are described by their probability distributions (*Figure 7*), then the preference done on the base of simplicity calculation could be given to point estimation. However, it is not the case for making constructive conclusions. Why? The answers that must follow are:

Firstly, it already has been said earlier that point estimation does not present adequate information about the future for decision makers and such kind of estimation in many cases could be misleading. One cannot feel psychological comfortability when using fault methodology;

Secondly, arrangement of information, which is necessary for decision making, especially standardized information must and will be presented by computerized profiles. User of information will have less chance to fail, but he or she should master himself in computer sciences.

Finally, contemporary calculators are already presenting probabilistically arranged information (like in 3-d and 4-d columns of *Table 1* and *Table 2*) and for the user there should not arise any problem except, maybe, for intellectual self perfection.

Final conclusions:

- Some non-adequacies of everyday practice in financial, investment, etc. decision preparing and making should be removed by adjusting decision making schemes and models with the probabilistic nature of future possibilities. In finance, for example, time value formulas should perform in accordance with probabilistic nature of interest rate and discount rate.
- The accumulated impact of amounting risk if it happens by increasing intensity of risk, or by enlarging time of accumulation manifestate a very one-sided tendency. It is a left-sided skewness of probability distribution density of the resulting outcome, independently on discounting or accumulative effects are prevailing in the processes under consideration. It means that risk presence generates an increasing share of unfavorable outcomes even if expectation remains the same. The result of these effects of presence of growing risk, compel investors to act like speculators and speculators to act like gamblers.

- Classical approach to risk and return commensurability should be completed by working out and employment of causal models for risk and return commensuration (*CMRRC*) which directly reflect dependency between return on investment and primary risk factors like return rate, exchange rate, discount rate etc. in explicitly or implicitly expressed functional form.
- The traditional investment appraisal methods are directly based on discounted outlays and inflows on an investment. Since point estimations do not reflect impact of contingencies on investment appraisal they could hardly be used for such a purpose.

  Imitative approach can help one to assess quantitatively direct impact of risk (variability) of discount rate on the probability distributions of *NPV* and *IRR*.
- Existing contemporary traditional territorial risk mapping systems for investors ought to be supplemented by risk value of money inform subsystems.

Table 1

Fragments of tables of discounted value of 1$ when interest rate behaviour in the future is given by point estimation and by probability distributions

	Discounted value of 1$	
Year	When point estimated interest rate, *i* = 10%	When forecasted probability distribution of interest rate in accordance with condition given on page 17
1	0,9090	N: m_1 = 0,9091 s_1 = 0,0004
2	0,8264	N: m_2 = 0,8264 s_2 = 0,0011
3	0,7513	N: m_3 = 0,7513 s_3 = 0,0020
4	0,6830	N: m_4 = 0,6831 s_4 = 0,0030
5	0,6209	N: m_5 = 0,6210 s_5 = 0,0042
6	0,5644	N: m_6 = 0,5646 s_6 = 0,0053
7	0,5131	N: m_7 = 0,5134 s_7 = 0,0065
8	0,4665	N: m_8 = 0,4668 s_8 = 0,0076
9	0,4240	LN: m_9 = 0,4245 s_9 = 0,0086
10	0,3855	LN: m_{10} = 0,3860 s_{10} = 0,0096
11	0,3504	LN: m_{11} = 0,3510 s_{11} = 0,0105
12	0,3186	LN: m_{12} = 0,3192 s_{12} = 0,0112
13	0,2896	LN: m_{13} = 0,2903 s_{13} = 0,0119
14	0,2633	LN: m_{14} = 0,2640 s_{14} = 0,0125
15	0,2393	LN: m_{15} = 0,2402 s_{15} = 0,0130
16	0,2176	LN: m_{16} = 0,2185 s_{16} = 0,0134
17	0,1978	LN: m_{17} = 0,1988 s_{17} = 0,0138
18	0,1798	LN: m_{18} = 0,1809 s_{18} = 0,0140
19	0,1635	LN: m_{19} = 0,1646 s_{19} = 0,0142
20	0,1486	LN: m_{20} = 0,1498 s_{20} = 0,0143
21	0,1351	LN: m_{21} = 0,1364 s_{21} = 0,0143
22	0,1228	LN: m_{22} = 0,1242 s_{22} = 0,0143
23	0,1116	LN: m_{23} = 0,1130 s_{23} = 0,0142
24	0,0391	LN: m_{24} = 0,1030 s_{24} = 0,0141
25	0,0922	LN: m_{25} = 0,0938 s_{25} = 0,0139
26	0,0839	LN: m_{26} = 0,0855 s_{26} = 0,0137
27	0,0762	LN: m_{27} = 0,0779 s_{27} = 0,0135
28	0,0693	LN: m_{28} = 0,0710 s_{28} = 0,0132
29	0,0630	LN: m_{29} = 0,0647 s_{29} = 0,0129
30	0,0573	LN: m_{30} = 0,0590 s_{30} = 0,0126

Table 2

Fragments of tables of accumulated value of 1$ when interest rate behavior in the future is given by point estimation and by probability distributions

	Accumulated value of 1$	
Year	Point estimated interest rate, *i* = 10%	Forecasted probability distribution of interest rate in accordance with condition given on page 17
1	1,1000	N: m_1 = 1,0999 s_1 = 0,0004
2	1,2100	N: m_2 = 1,2099 s_2 = 0,0016
3	1,3310	N: m_3 = 1,3308 s_3 = 0,0036
4	1,4641	N: m_4 = 1,4638 s_4 = 0,0066
5	1,6105	N: m_5 = 1,6101 s_5 = 0,0109
6	1,7715	N: m_6 = 1,7710 s_6 = 0,0168
7	1,9487	N: m_7 = 1,9479 s_7 = 0,0246
8	2,1435	N: m_8 = 2,1426 s_8 = 0,0349
9	2,3579	N: m_9 = 2,3566 s_9 = 0,0480
10	2,5937	N: m_{10} = 2,5921 s_{10} = 0,0645
11	2,8531	N: m_{11} = 2,8512 s_{11} = 0,0851
12	3,1384	LN: m_{12} = 3,1363 s_{12} = 0,1107
13	3,4522	LN: m_{13} = 3,4499 s_{13} = 0,1421
14	3,7974	LN: m_{14} = 3,7950 s_{14} = 0,1804
15	4,1772	LN: m_{15} = 4,1749 s_{15} = 0,2268
16	4,5949	LN: m_{16} = 4,5930 s_{16} = 0,2828
17	5,0544	LN: m_{17} = 5,0534 s_{17} = 0,3501
18	5,5599	LN: m_{18} = 5,5603 s_{18} = 0,4305
19	6,1159	LN: m_{19} = 6,1185 s_{19} = 0,5265
20	6,7275	LN: m_{20} = 6,7335 s_{20} = 0,6405
21	7,4002	LN: m_{21} = 7,4111 s_{21} = 0,7757
22	8,1402	LN: m_{22} = 8,1578 s_{22} = 0,9354
23	8,9543	LN: m_{23} = 8,9810 s_{23} = 1,1238
24	9,8497	LN: m_{24} = 9,8888 s_{24} = 1,3455
25	10,8347	LN: m_{25} = 10,8900 s_{25} = 1,6060
26	11,9181	LN: m_{26} = 11,9948 s_{26} = 1,9115
27	13,1099	LN: m_{27} = 13,2142 s_{27} = 2,2692
28	14,4209	LN: m_{28} = 14,5606 s_{28} = 2,6876
29	15,8630	LN: m_{29} = 16,0478 s_{29} = 3,1762
30	17,4494	LN: m_{30} = 17,6913 s_{30} = 3,7464

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